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TRENDS IN THE GLOBULAR CLUSTER LUMINOSITY FUNCTION OF EARLY-TYPE GALAXIES[1]

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ABSTRACT

We present results from a study of the globular cluster luminosity function (GCLF) in a sample of 89 early-type galaxies observed as part of the ACS Virgo Cluster Survey. Using a Gaussian parameterization of the GCLF, we find a highly significant correlation between the GCLF dispersion, σ, and the galaxy luminosity, $M_{B,\,\mathrm{gal}}$, in the sense that the GC systems in fainter galaxies have narrower luminosity functions. The GCLF dispersions in the Milky Way and M31 are fully consistent with this trend, implying that the correlation between σ and galaxy luminosity is more fundamental than older suggestions that GCLF shape is a function of galaxy Hubble type. We show that the σ-$M_{B,\,\mathrm{gal}}$ relation results from a bona fide narrowing of the distribution of (logarithmic) cluster masses in fainter galaxies. We further show that this behavior is mirrored by a steepening of the GC mass function for relatively high masses, $\mathcal{M} \gtrsim 3 \times 10^5\ \mathcal{M}_\odot$, a mass regime in which the shape of the GCLF is not strongly affected by dynamical evolution over a Hubble time. We argue that this trend arises from variations in initial conditions and requires explanation by theories of cluster formation. Finally, we confirm that in bright galaxies the GCLF "turns over" at the canonical mass scale of $\mathcal{M}_{\mathrm{TO}} \simeq 2 \times 10^5\ \mathcal{M}_\odot$. However, we find that $\mathcal{M}_{\mathrm{TO}}$ scatters to lower values [$\approx$(1–2) $\times\ 10^5\ \mathcal{M}_\odot$] in galaxies fainter than $M_{B,\,\mathrm{gal}} \gtrsim -18.5$, an important consideration if the GCLF is to be used as a distance indicator for dwarf ellipticals.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: star clusters — globular clusters: general

1. INTRODUCTION

The luminosity function of globular clusters (GCs) represents one of the most remarkable features of these stellar systems. The distribution of GC magnitudes, commonly referred to as the GC luminosity function (GCLF), shows a turnover, or peak, at $M_V \simeq -7.5$ mag, corresponding to a mass of $\mathcal{M} \simeq 2 \times 10^5\ \mathcal{M}_\odot$. Observations have shown that this turnover is nearly invariant across and within galaxies, prompting its widespread use as a distance indicator (see, e.g., Harris 2001). Accounting for this nearly universal mass scale remains an open problem for theories of GC formation and evolution. It follows that establishing whether or not the GCLF as a whole is universal (i.e., whether its overall form depends on host galaxy properties) can help guide and constrain theories for the formation and evolution of galaxies and GC systems.

In this Letter, we present results from a study of the GCLFs of 89 early-type galaxies observed by *HST* as part of the ACS Virgo Cluster Survey (ACSVCS; Côté et al. 2004). We find the clearest evidence to date of a correlation between the width (i.e., Gaussian dispersion) of the GCLF and the luminosity of the host galaxy; we also show that there is some downward scatter in the mass scale of the GCLF turnover in galaxies fainter than $M_{B,\,\mathrm{gal}} \gtrsim -18.5$. Focusing on the observed steepening of the GCLF at the bright (high-mass) end in the faint galaxies, we argue that this behavior was probably imprinted at the time of GC formation. A more detailed discussion of the whole GCLF, including the faint (low-mass) end and the role that long-term dynamical evolution plays in that regime, is deferred to a subsequent paper (Jordán et al. 2006, hereafter J06). That paper presents our data in full and gives details of our analysis techniques, including modeling of the GCLFs with a new, non-Gaussian, physically motivated fitting function.

2. OBSERVATIONS AND ANALYSIS

One hundred early-type galaxies in the Virgo Cluster were observed in the ACSVCS (Côté et al. 2004). Each galaxy was imaged for 750 s in the F475W bandpass ($\simeq$Sloan *g*) and for 1210 s in F850LP ($\simeq$Sloan *z*). Reductions were performed as described in Jordán et al. (2004). In what follows, we use *g* and *z* as shorthand to refer to the F475W and F850LP filters.

One of the main scientific objectives of the ACSVCS is the study of GC systems, and thus we have developed methods to (1) discard foreground stars and background galaxies from the totality of observed sources around each target galaxy in the survey and (2) estimate the level of residual fore- and background contamination in the remaining sources designated as candidate GCs. These procedures are described and illustrated by Peng et al. (2006a; see their § 2.2 and Fig. 1), and discussed in detail in the GCLF context in J06. In the latter paper, we also examine the effects of using alternate selection criteria to define GC samples, and show that the results presented here are fully robust against such subtleties.

Of the 100 galaxies in the ACSVCS, we restrict our analysis to those that have more than five GCs, as estimated by subtracting the total number of expected contaminants from the

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full list of GC candidates for each galaxy. We additionally eliminate two galaxies for which we were unable to obtain useful measurements of the GCLF parameters. This leaves a final sample of 89 galaxies that are studied here and in J06.

Also as part of the ACSVCS, we have measured the distances to 84 of our target galaxies using the method of surface brightness fluctuations (SBF; Mei et al. 2006). We use these SBF distances to transform the observed GC and galaxy magnitudes into absolute ones whenever possible. For those galaxies lacking an SBF distance, we adopt the mean distance modulus to the Virgo Cluster: $\langle(m-M)_0\rangle = 31.09$ mag or $\langle D\rangle = 16.5$ Mpc (see Mei et al. 2005, 2006).

We use an approach similar to that of Secker & Harris (1993) to characterize the GCLFs; parametric models are fitted to the observed luminosity functions via a maximum likelihood method that takes into account photometric errors, incompleteness, and the luminosity function of contaminants. Full technical details are given in J06, where we consider two parametric models for the GCLF. The first, on which this Letter will focus, is the standard Gaussian distribution,

$$dN/dz = N_{\rm tot}(2\pi\sigma_z^2)^{-1/2} \exp\,[-(z-\mu_z)^2/2\sigma_z^2]. \qquad (1)$$

The second is a simple analytical modification of a Schechter (1976) function designed to account for the effects of cluster evaporation (two-body relaxation) on a GC mass function that is assumed to have initially resembled that of the young clusters forming today in local mergers and starbursts. Full details on these two models are given in J06, where we fit each of them to the separate *g*- and *z*-band GCLFs of our 89 program galaxies. In this Letter we present only the results of Gaussian fits to the *z*-band GCLFs.

3. RESULTS

Figure 1 shows our main result: GCLFs are narrower in lower luminosity galaxies. The straight line in this plot of Gaussian dispersion against absolute galaxy magnitude shows the least-squares fit

$$\sigma_z = (1.12 \pm 0.01) - (0.093 \pm 0.006)(M_{B,\,\rm gal} + 20). \qquad (2)$$

It has been reported before that the GCLFs in lower luminosity galaxies tend to show somewhat lower dispersions (e.g., Kundu & Whitmore 2001). However, the size and homogeneity of the ACSVCS data set make this the most convincing demonstration to date of a continuous trend in GCLF shape over a range of $\gtrsim$400 in galaxy luminosity. Monte Carlo simulations and alternate constructions of GCLF samples show that the observed decrease in dispersion is *not* an artifact of small-number statistics in the faint galaxies (J06).

Past investigations have pointed to a dependence of the GCLF dispersion on the Hubble type of the GC host galaxies (e.g., Harris 1991). Figure 1 includes data points at the location of the bulge magnitude and GCLF dispersion of the Milky Way (*large star*) and M31 (*large triangle*). Since both systems fall comfortably on the relation defined by our data for early-type galaxies, we conclude that the underlying fundamental correlation is one between σ and $M_{B,\,\rm gal}$, rather than between σ and Hubble type.

A natural question at this point is whether the observed trend in GCLF dispersion with galaxy magnitude implies a similar trend in the GC mass function. This is not a foregone conclusion for the following reason: GC systems are known to have systematically redder and broader (or more strongly bimodal) color distributions in brighter galaxies than in faint ones (see, e.g., Peng et al. 2006a). Equivalently, GCs in giant galaxies are more metal-rich, on average, and have larger dispersions in [Fe/H] than those in low-mass dwarfs. Since cluster mass-to-light ratios, Υ, are functions of [Fe/H] in general, it is conceivable that the average GC Υ could change systematically in going from bright galaxies to fainter ones, and that the spread of Υ-values within a single GC system could also vary systematically as a function of galaxy magnitude. The possibility then exists that narrower GCLFs for faint galaxies might result from these systematics in Υ combined with a more nearly invariant spread in GC masses. We can show easily, however, that this is not the case.

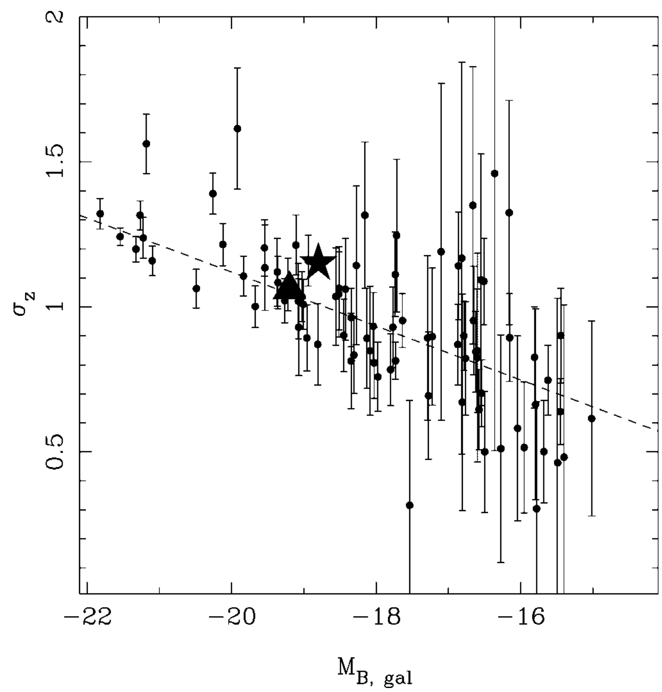

FIG. 1.—Gaussian dispersion, σ_z, vs. galaxy, $M_{B,\,\rm gal}$, for the *z*-band GCLFs of 89 ACSVCS galaxies. The GCLF width varies systematically, being narrower in fainter galaxies. The two anomalously high points at $M_{B,\,\rm gal} = -21.2$ and -19.9 correspond to the galaxies VCC 798 and VCC 2095, both of which have large excesses of faint, diffuse clusters (Peng et al. 2006b). The large star is plotted at the spheroid luminosity (de Vaucouleurs & Pence 1978) and GCLF dispersion (Harris 2001) of the Milky Way. The large triangle marks the bulge luminosity (Kent 1989) and GCLF dispersion (Harris 2001) of M31.

The systematics in Υ versus [Fe/H] just mentioned are also a function of wavelength. In bluer filters, such as *B*, *V*, or *g*, mass-to-light ratios of old stellar systems do change significantly (increasing by factors of 2 or more) in going from cluster metallicities [Fe/H] $\lesssim -2$ to [Fe/H] $= 0$, typical of GCs. But at the much redder wavelengths of our *z*-band data ($\lambda_{\rm pivot} \simeq$ 9055 Å; Sirianni et al. 2005), this strong metallicity dependence almost completely disappears. We have used the PEGASE population synthesis model of Fioc & Rocca-Volmerange (1997) to compute Υ_z as a function of metallicity for clusters with a Kennicutt (1983) stellar initial mass function (IMF) and various fixed ages τ. For $\tau = 13$ Gyr, we find that $\Upsilon_z \simeq 1.6\ \mathcal{M}_\odot\ L_\odot^{-1}$ at an extreme [Fe/H] $= -2.3$, decreasing to a minimum of $\Upsilon_z \simeq 1.5\ \mathcal{M}_\odot\ L_\odot^{-1}$ at [Fe/H] $\simeq -0.7$, and then increasing slightly to $\Upsilon_z = 1.7\ \mathcal{M}_\odot\ L_\odot^{-1}$ at [Fe/H] $= 0$. In other words, we always have $\Upsilon_z \approx 1.6 \pm 0.1$ for any of the globular clusters in any of our sample galaxies, no matter how red or blue the clusters are, or how broad or narrow the GC color/metallicity distribution is. Comparably small ranges of Υ_z result if younger GC ages or different reasonable stellar IMFs are assumed.

The effect of variations in mass-to-light ratio on the width of the GCLF at near-infrared wavelengths is therefore completely negligible. From Figure 1, we have that $\sigma(\log L_z) = \sigma_z/2.5 \gtrsim 0.2$ in our galaxies, whereas the discussion above implies that $\Upsilon_z(\max)/\Upsilon_z(\min) \sim 1.13$ for our GCs, such that the dispersion of mass-to-light ratios in any one system is always $\sigma(\log \Upsilon_z) < 0.055$ at an absolute maximum. The intrinsic dispersion of logarithmic GC masses, $\sigma(\log \mathcal{M}) = [\sigma^2(\log L_z) - \sigma^2(\log \Upsilon_z)]^{1/2}$, is thus never more than ~4% different from the observed $\sigma(\log L_z)$. We conclude, unavoidably, that the steady decrease of σ_z by more than 50% from the brightest giants to the faintest dwarfs in Figure 1 is an accurate reflection of just such a trend in the intrinsic GC mass distributions.

We now turn our attention to the GCLF turnover magnitude. The top panel of Figure 2 shows the mean GC absolute magnitude μ_z from the Gaussian fits to our GCLFs versus host galaxy $M_{B,\,\mathrm{gal}}$. The horizontal line in this plot is drawn at a level typical of galaxies brighter than $M_{B,\,\mathrm{gal}} \lesssim -18.5$: $\mu_z = -8.4$. Given a typical $\Upsilon_z \simeq 1.5\ \mathcal{M}_\odot\ L_\odot^{-1}$ in these galaxies (for GC ages 13 Gyr and an average [Fe/H] ≈ -1), this corresponds to a cluster mass scale of $\mathcal{M}_{\mathrm{TO}} \simeq 2.2 \times 10^5\ \mathcal{M}_\odot$. Estimates of the z-band GCLF turnovers in the Milky Way and M31 are shown by the large star and large triangle, as in Figure 1. In the bottom panel of Figure 2, we plot the turnover masses $\mathcal{M}_{\mathrm{TO}}$ obtained from the fitted μ_z using PEGASE model mass-to-light ratios. As we have discussed, z-band luminosities are very good proxies for total GC masses, so this graph is essentially a mirror image of the one above it.

Figure 2 shows that there is no strong or systematic variation in μ_z or $\mathcal{M}_{\mathrm{TO}}$ to match that seen for σ_z (Fig. 1). Nevertheless, there is a clear tendency for the GCLF turnovers of galaxies fainter than $M_{B,\,\mathrm{gal}} \gtrsim -18.5$ to scatter to somewhat fainter (less massive) values than is typical of the bright giants. The difference in mass is a factor of ≈1.5 on average, but it ranges apparently randomly, from a factor of 1 (i.e., no difference) up to factors slightly greater than 2 in some cases. Note that there is a healthy mix of E and S0 or dE and dS0 galaxies at all magnitudes in our ACSVCS sample (see Table 1 of Côté et al. 2004). We find no tendency for any particular Hubble type to scatter preferentially away from $\mu_z = -8.4$ or $\mathcal{M}_{\mathrm{TO}} = 2.2 \times 10^5\ \mathcal{M}_\odot$ in Figure 2.

The lower mean value for $\mathcal{M}_{\mathrm{TO}}$ at faint $M_{B,\,\mathrm{gal}}$ clearly can impact the use of the GCLF as a standard candle for dwarf galaxies. On the other hand, the effect is wavelength-dependent. Publicly available codes such as PEGASE can be easily used to show that in bluer bandpasses such as g (or the closely related V, which is more standard for such studies), the slight decrease we find for the average GC turnover mass in fainter galaxies is balanced by a comparable decrease in the typical GC mass-to-light ratio (because of the lower cluster metallicities), so that the mean turnover magnitude does not vary as in the z band. We have also confirmed this directly from our own ACSVCS data. In J06, we obtain plots analogous to Figures 1 and 2 from fits to the g-band GCLFs of our galaxies. The results fully support all of our conclusions here. It is particularly worth noting that we find a relation identical to equation (2) for the dependence of g-band GCLF dispersion on parent galaxy luminosity.

4. DISCUSSION

An obvious question prompted by Figure 1 is whether the correlation between σ_z and $M_{B,\,\mathrm{gal}}$ was established at the time of cluster formation or built up afterward as GCLFs were modified by the dynamical destruction of GCs over a Hubble time. We favor the first interpretation.

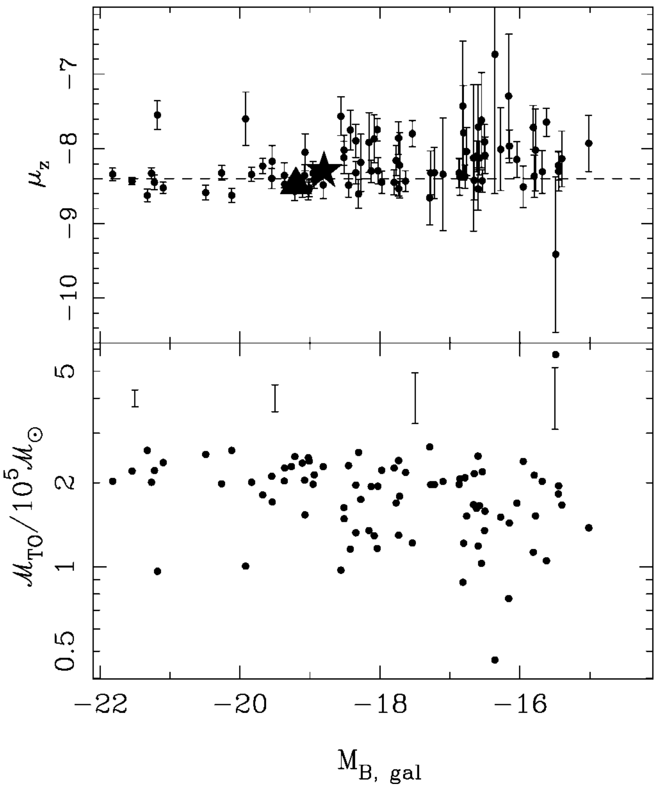

FIG. 2.—*Top*: GCLF turnover magnitude (absolute mean μ_z) vs. galaxy magnitude, $M_{B,\,\mathrm{gal}}$, from Gaussian fits to 89 z-band GCLFs in the ACSVCS. The outlying points at $M_{B,\,\mathrm{gal}} = -21.2$ and -19.9 are VCC 798 and VCC 2095, respectively, which have large excesses of faint, diffuse clusters (Peng et al. 2006b). The large star and triangle show μ_z values for the Milky Way and M31, respectively, estimated from their V-band peaks (Harris 2001) by applying an average $(V - z)$ color estimated from the PEGASE population synthesis code (Fioc & Rocca-Volmerange 1997). *Bottom*: Turnover mass $\mathcal{M}_{\mathrm{TO}}$ corresponding to the fitted μ_z, obtained by applying an average GC Υ_z computed for each galaxy using the PEGASE model. Typical error bars on $\mathcal{M}_{\mathrm{TO}}$ as a function of galaxy magnitude are indicated.

Star clusters can be destroyed over gigayear timescales as a result of mass loss driven by stellar evolution, dynamical friction, gravitational shocks, and internal two-body relaxation (evaporation) processes that have been studied in detail by several groups. Recent discussions, centered specifically on how these affect the GCLF, can be found in Fall & Zhang (2001) and Vesperini (2000, 2001). Fall & Zhang, in particular, show that, while stellar evolution and gravitational shocks certainly deplete the total number of GCs in a galaxy, they do not significantly alter the overall shape of the GCLF. Evaporation, on the other hand, *can* change the shape of the GCLF, but significantly so only for cluster masses $\mathcal{M} \lesssim (2\text{–}3) \times 10^5\ \mathcal{M}_\odot$, i.e., below the typical GCLF turnover mass.

In the theoretical treatments of Fall & Zhang, Vesperini, and many others, the evaporation rate is independent of cluster mass, which ultimately drives the low-mass side of the GCLF to a universal shape: a simple exponential $dN/dz \propto 10^{-0.4z}$ for the number of GCs per unit magnitude fainter than the turnover (equivalent to a flat distribution for the number of GCs per unit linear luminosity or mass). But fitting a Gaussian model to the GCLF, as we have done here, tacitly assumes that the distribution is symmetric. The results in Figure 1 might, therefore, seem to imply that both the bright side *and* the faint side of the GCLF become progressively steeper in fainter galaxies. However, various observational uncertainties make it difficult to determine precisely the form of the faintest tail of the GCLF. Thus, in J06 we show that good fits to our GCLFs can also be obtained using an alternate model with a universal exponential shape at magnitudes fainter than the turnover and that the downward scatter in $\mathcal{M}_{\rm TO}$ for faint galaxies persists in such a model and so is not an artifact of any assumed Gaussian symmetry. Here we concern ourselves only with the brighter half of the GCLF, which is observationally better defined.

We have performed maximum likelihood fits of exponential models $dN/dz \propto 10^{0.4(\beta_z-1)z}$ (corresponding to power-law mass distributions, $dN/d\mathcal{M} \propto \mathcal{M}^{-\beta_z}$) to the GCLFs at absolute magnitudes $-8.7 \gtrsim z \gtrsim -10.8$ (cluster masses $\simeq 3 \times 10^5$–$2 \times 10^6\ \mathcal{M}_\odot$) in 66 of our galaxies. Such distributions accurately describe the bright sides of giant galaxy GCLFs (Harris & Pudritz 1994; Larsen et al. 2001), and with $\beta_z \simeq 2$, they also give good matches to the mass functions of young star clusters in nearby mergers and starbursts (Zhang & Fall 1999).

Figure 3 shows the results of this exercise. There is a clear steepening in the power-law exponent, from $\beta_z \simeq 1.8$ in bright galaxies to $\beta_z \simeq 3$ in the faintest systems. However the faint side of the GCLF behaves in detail, the bright side alone suggests that smaller galaxies were unable to form very massive clusters in the same *relative* proportions as giant galaxies.

A potential complication here is dynamical friction. A cluster of mass $\mathcal{M}$ on an orbit of radius r in a galaxy with circular speed V_c will spiral into the center of the galaxy on a timescale $\tau_{\rm df} \propto \mathcal{M}^{-1} r^2 V_c$ (Binney & Tremaine 1987). In the Milky Way and larger galaxies, $\tau_{\rm df} > 13$ Gyr for all but the very most massive clusters at small radii, and thus dynamical friction does not significantly affect their GCLFs (e.g., Fall & Zhang 2001). In dwarfs with low V_c, however, $\tau_{\rm df}$ can be interestingly short for smaller GCs at larger r, suggesting, perhaps, that the process might significantly deplete the bright side of the GCLF in small galaxies and contribute to the type of trend seen in Figure 3. However, Vesperini (2000, 2001) has modeled the GCLF evolution over a Hubble time in galaxies with a wide range of mass, and his results strongly suggest that dynamical friction does *not* suffice to explain our observations. In particular, the widths of the Gaussian GCLFs in his models do not decrease, even in dwarf galaxies, to anywhere near the extent seen in the data. Thus, any significant galaxy-to-galaxy variations in the shape of the GCLF above the turnover mass probably reflect initial conditions (see J06 for further discussion).

In summary, the gradual narrowing of the GCLF as a function of galaxy luminosity, or the steepening of the mass distribution above the classic turnover point, presents a new constraint for theories of GC formation and evolution. In our view, it is the cluster formation process in particular that is likely to be most relevant to the observed behavior at the high-mass end of the GCLF. Exactly what factors might lead to more massive galaxies forming massive clusters in greater relative numbers, is an open question of some interest.

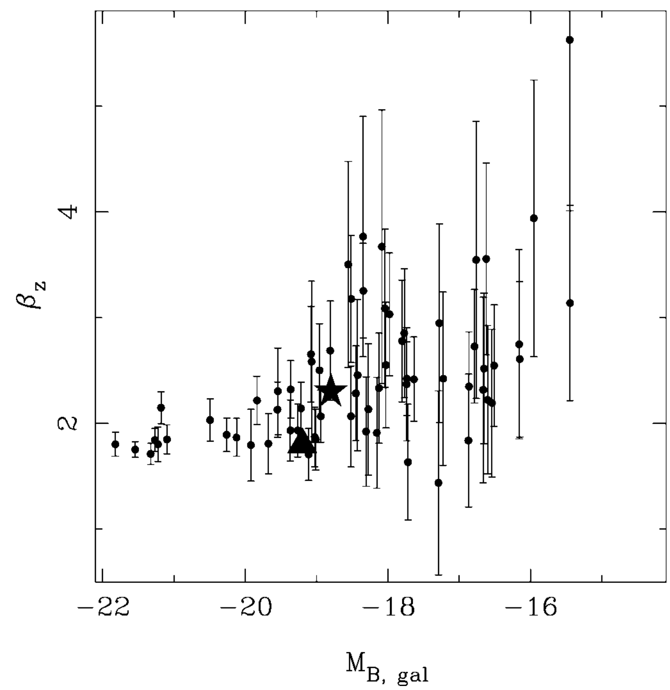

FIG. 3.—Slope of the power law that best fits our z-band GCLF data, β_z, for masses $3 \times 10^5 \lesssim (\mathcal{M}/\mathcal{M}_\odot) \lesssim 2 \times 10^6$, plotted against host galaxy absolute magnitude, $M_{B,\rm gal}$. The large star and triangle show β-values for the Milky Way and M31, respectively, measured in the same mass regime using the data from Harris (1996) and Reed et al. (1994) assuming a V-band mass-to-light ratio $M/L_V = 2$. The bright side of the GCLF is steeper in fainter galaxies.

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REFERENCES

Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press)
Côté, P., et al. 2004, ApJS, 153, 223
de Vaucouleurs, G., & Pence, W. D. 1978, AJ, 83, 1163
Fall, S. M., & Zhang, Q. 2001, ApJ, 561, 751
Fioc, M., & Rocca-Volmerange, B. 1997, A&A, 326, 950
Harris, W. E. 1991, ARA&A, 29, 543
———. 1996, AJ, 112, 1487
———. 2001, in Star Clusters, ed. L. Labhardt & B. Binggeli (Berlin: Springer), 223
Harris, W. E., & Pudritz, R. E. 1994, ApJ, 429, 177
Jordán, A., et al. 2004, ApJS, 154, 509
———. 2006, ApJS, submitted (J06)
Kennicutt, R. C., Jr. 1983, ApJ, 272, 54
Kent, S. M. 1989, AJ, 97, 1614
Kundu, A., & Whitmore, B. C. 2001, AJ, 121, 2950
Larsen, S. S., Brodie, J. P., Huchra, J. P., Forbes, D. A., & Grillmair, C. J. 2001, AJ, 121, 2974
Mei, S., et al. 2005, ApJ, 625, 121
———. 2006, ApJ, in press
Peng, E. W., et al. 2006a, ApJ, 639, 95
———. 2006b, ApJ, 639, 838
Reed, L. G., Harris, G. L. H., & Harris, W. E. 1994, AJ, 107, 555
Schechter, P. L. 1976, ApJ, 203, 297
Secker, J., & Harris, W. E. 1993, AJ, 105, 1358
Sirianni, M., et al. 2005, PASP, 117, 1049
Vesperini, E. 2000, MNRAS, 318, 841
———. 2001, MNRAS, 322, 247
Zhang, Q., & Fall, S. M. 1999, ApJ, 527, L81